Statistical Estimation Under Group Actions, with Applications to Cryo-Electron Microscopy

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In many problems in computer vision, robotics and image/signal processing, we wish to recover latent variables from observations suffering unknown shifts or rotations. One example is cryo-electron microscopy (cryo-EM), recognized by the Nobel Prize 2017 in Chemistry. Here the challenge is to estimate the 3D structure of a protein from many, very noisy 2D images taken at unknown viewing directions.

In this talk, I will place cryo-EM reconstruction inside a framework for statistical estimation under noisy group actions. I will prove a tight relation between the sample complexity for statistical learning under noisy group actions and the invariant theory of the underlying symmetry group. On the algebra side, this motivates apparently new questions in invariant theory -- to which we offer partial algorithmic answers in general. As for the cryo-EM case, we will sketch a novel ab initio 3D reconstruction algorithm based upon tensor decomposition, which is both sample and computationally efficient -- at least under model assumptions.